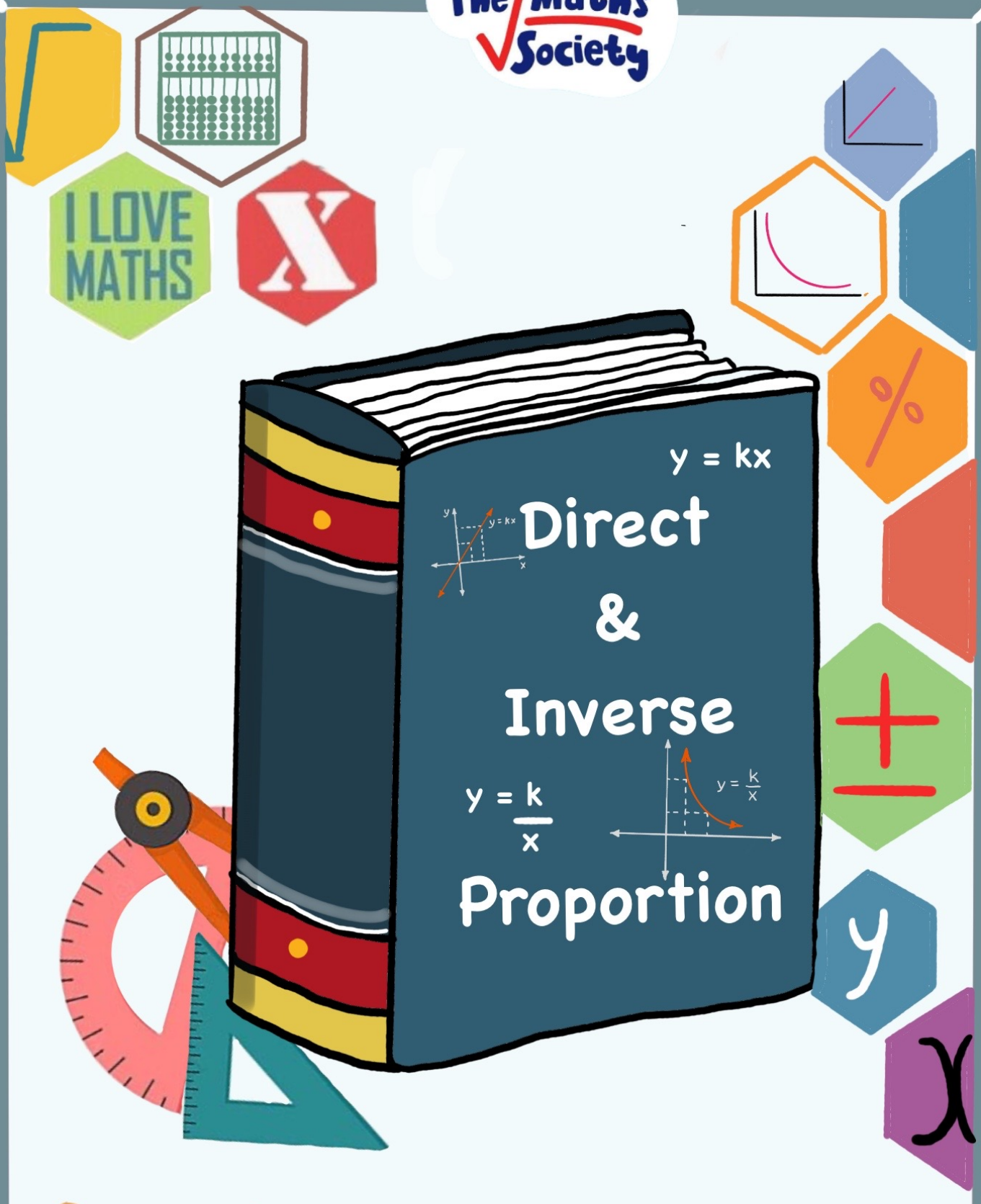
The Maths Society



Question 1

y is directly proportional to the positive square root of x.
When $x = 9$, $y = 12$.

[3]

Find y when $x = \frac{1}{4}$.

$y \propto \sqrt{x}$
$y = k\sqrt{x}$
$12 = k\sqrt{9}$
$12 = 3k$
$k = 4$

$y = 4\sqrt{x}$
$y = 4 \times \frac{1}{2} = 2$

Question 2

V is directly proportional to the cube of $(r + 1)$.
When $r = 1$, $V = 24$.

Work out the value of V when $r = 2$. [3]

$V \propto (r+1)^3$
$V = k(r+1)^3$
$24 = k \times 8$
$k = 3$

$V = 3(r+1)^3$
$V = 3(3)^3$
$= 81$

Question 3

y is directly proportional to the square of $(x - 1)$.
$y = 63$ when $x = 4$.

[3]

Find the value of y when $x = 6$.

$y \propto (x-1)^2$
$y = k(x-1)^2$
$63 = k \times 9$
$k = 7$

$y = 7(x-1)^2$
$y = 7 \times 25$
$= 175$

Question 4

y is inversely proportional to $(x + 2)^2$.
When $x = 1$, $y = 2$.

[2]

Find y in terms of x.

$y \propto \frac{1}{(x+2)^2}$
$y = \frac{k}{(x+2)^2}$
$2 = \frac{k}{9}$
$k = 18$
$y = \frac{18}{(x+2)^2}$

Question 5

p is inversely proportional to the square of $(q + 4)$.
$p = 2$ when $q = 2$.

[3]

Find the value of p when $q = -2$.

$p \propto \frac{1}{(q+4)^2}$

$p = \frac{k}{(q+4)^2}$

$2 = \frac{k}{36}$

$k = 72$

$p = \frac{72}{(-2+4)^2} = \frac{72}{4}$

$= 18$

Question 6

The number of hot drinks sold in a café decreases as the weather becomes warmer.

What type of correlation does this statement show?

negative correlation

[1]

Question 7

x varies directly as the cube root of y.
$x = 6$ when $y = 8$.

[3]

Find the value of x when $y = 64$.

$x \propto \sqrt[3]{y}$

$x = k\sqrt[3]{y}$

$6 = k \times 2$

$k = 3$

$x = 3\sqrt[3]{y}$

$= 3 \times 4$

$= 12$

Question 8

y varies directly with $\sqrt{x+5}$.

$y = 4$ when $x = -1$.

[3]

Find y when $x = 11$.

$y \propto \sqrt{x+5}$

$y = k\sqrt{x+5}$

$4 = k \times 2$

$k = 2$

$y = 2 \times 4$

$= 8$

Question 9

The cost of a circular patio, $\$C$, varies as the square of the radius, r metres.

$C = 202.80$ when $r = 2.6$.

Calculate the cost of a circular patio with $r = 1.8$. [3]

$C \propto r^2$

$C = kr^2$

$202.8 = k(2.6)^2$

$k = 30$

$C = 30 \times (1.8)^2$

$= 97.2$

Question 1

y varies inversely as $(x + 5)$.

$y = 6$ when $x = 3$.

Find y when $x = 7$. [3]

$y \propto \frac{1}{x+5}$

$6 = \frac{k}{8}$

$k = 48$

$y = \frac{48}{12} = 4$

Question 2

w varies inversely as the square root of x.
When $x = 4$, $w = 4$.

Find w when $x = 25$. [3]

$w \propto \frac{1}{\sqrt{x}}$

$w = \frac{k}{\sqrt{x}}$

$4 = \frac{k}{2}$

$k = 8$

$w = \frac{8}{\sqrt{25}}$

$w = \frac{8}{5}$

Question 3

y varies as the cube root of $(x + 3)$.
When $x = 5$, $y = 1$.

Find the value of y when $x = 340$. [3]

$y \propto \sqrt[3]{x+3}$

$y = k(x+3)^{1/3}$

$1 = k(5+3)^{1/3}$

$1 = k(8)^{1/3}$

$1 = k(2^3)^{1/3}$

$1 = k \cdot 2$

$k = \frac{1}{2}$

$y = \frac{1}{2}(340+3)^{1/3}$

$= \frac{1}{2}(343)^{1/3}$

$= \frac{1}{2}(7^3)^{1/3}$

$= \frac{7}{2}$

Question 4

The speed, v, of a wave is inversely proportional to the square root of the depth, d, of the water.

$v = 30$ when $d = 400$.

Find v when $d = 25$. [3]

$v \propto \frac{1}{\sqrt{d}}$

$v = \frac{k}{\sqrt{d}}$

$30 = \frac{k}{\sqrt{400}}$

$30 = \frac{k}{20}$

$k = 600$

$v = \frac{600}{\sqrt{25}}$

$= \frac{600}{5}$

$= 120$

Question 5

m varies directly as the cube of x.

$m = 200$ when $x = 2$.

Find m when $x = 0.4$. [3]

$m \propto x^3$

$m = kx^3$

$200 = k(2)^3$

$200 = k \times 8$

$k = \frac{200}{8}$

$k = 25$

$m = 25\ (0.4)^3$

$= 25\ (0.064)$

$= 1.54$

Question 6

y is inversely proportional to x^3.

$y = 5$ when $x = 2$.

Find y when $x = 4$. [3]

$y \propto \frac{1}{x^3}$

$y = \frac{k}{x^3}$

$5 = \frac{k}{2^3}$

$5 = \frac{k}{8}$

$k = 40$

$y = \frac{40}{4^3}$

$= \frac{40}{64}$

$= \frac{5}{8}$

Question 7

The mass, m, of a sphere varies directly with the **cube** of its radius, r.

$m = 160$ when $r = 2$.

Find m when $r = 5$. [3]

$m \propto r^3$

$m = kr^3$

$160 = k(2)^3$

$160 = k \cdot 8$

$k = \frac{160}{8}$

$k = 20$

$m = 20(5)^3$

$= 20 \times 125$

$= 2500$

Question 8

The electrical resistance, R, of a length of cylindrical wire varies inversely as the square of the diameter, d, of the wire.
$R = 10$ when $d = 2$.

Find R when $d = 4$. [3]

$R \propto \frac{1}{d^2}$

$R = \frac{k}{d^2}$

$10 = \frac{k}{2^2}$

$10 = \frac{k}{4}$

$k = 40$

$R = \frac{40}{4^2}$

$= \frac{40}{16}$

$= \frac{5}{2}$

Question 9

The mass, m, of an object varies directly as the **cube** of its length, l.

$m = 250$ when $l = 5$.

Find m when $l = 7$. [3]

$m \propto l^3$

$m = kl^3$

$250 = k(5)^3$

$250 = 125k$

$k = \frac{250}{125}$

$k = 2$

$m = 2(7)^3$

$= 2 \times 343$

$= 686$

Question 10

y varies inversely as the square root of x.
When $x = 9$, $y = 6$.

Find y when $x = 36$. [3]

$y \propto \frac{1}{\sqrt{x}}$

$y = \frac{k}{\sqrt{x}}$

$6 = \frac{k}{\sqrt{9}}$

$6 = \frac{k}{3}$

$k = 18$

$y = \frac{18}{\sqrt{36}}$

$= \frac{18}{6}$

$= 3$

Question 11

y is **inversely** proportional to x^2.
When $x = 4$, $y = 3$.

Find y when $x = 5$. [3]

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2}$

$3 = \frac{k}{4^2}$

$3 = \frac{k}{16}$

$k = 48$

$y = \frac{48}{5^2}$

$= \frac{48}{25}$

Question 12

y varies directly as the square of $(x - 3)$.
$y = 16$ when $x = 1$.

Find y when $x = 10$. [3]

$y \propto (x-3)^2$

$y = k(x-3)^2$

$16 = k(1-3)^2$

$16 = 4k$

$k = 4$

$y = 4(10-3)^2$

$= 4 \times 49$

$= 196$

Question 1

The periodic time, T, of a pendulum varies directly as the square root of its length, l.
$T = 6$ when $l = 9$.

Find T when $l = 25$. [3]

$T \propto \sqrt{l}$

$T = k\sqrt{l}$

$6 = k\sqrt{9}$

$6 = 3k$

$k = 2$

$T = 2\sqrt{25}$

$= 2 \times 5$

$= 10$

Question 2

Seismic shock waves travel at speed v through rock of density d.
v varies inversely as the square root of d.

$v = 3$ when $d = 2.25$.

Find v when $d = 2.56$. [3]

$v \propto \frac{1}{\sqrt{d}}$

$v = \frac{k}{\sqrt{d}}$

$3 = \frac{k}{\sqrt{2.25}}$

$3 = \frac{k}{1.5}$

$k = 4.5$

$v = \frac{4.5}{\sqrt{2.56}}$

$= \frac{4.5}{1.6}$

$= \frac{45}{16}$

Question 3

The force, F, between two magnets varies inversely as the square of the distance, d, between them.

$F = 150$ when $d = 2$.

Calculate F when $d = 4$. [3]

$F \propto \frac{1}{d^2}$

$F = \frac{k}{d^2}$

$150 = \frac{k}{2^2}$

$k = 150 \times 4$

$= 600$

$F = \frac{600}{4^2}$

$= \frac{600}{16} = 37.5$

Question 4

The time, t, for a pendulum to swing varies directly as the square root of its length, l.
When $l = 9$, $t = 6$.

(a) Find a formula for t in terms of l. [2]

$t \propto \sqrt{l}$ | $6 = k\sqrt{9}$ | $t = 2\sqrt{l}$

$t = k\sqrt{l}$ | $6 = 3k$ | $k = 2$

(b) Find t when $l = 2.25$. [1]

$t = 2\sqrt{2.25} = 2 \times 1.5 = 3$

Question 5

The volume of a solid varies directly as the cube of its length.
When the length is 3 cm, the volume is $108\,cm^3$.

Find the volume when the length is 5 cm. [3]

$V \propto l^3$ | $108 = k(3)^3$ | $V = 4 \times (5)^3$

$V = kl^3$ | $108 = 27k$ | $= 4 \times 125$

$k = 4$ | $= 500$

Question 6

The resistance, R, of an object being towed through the water varies directly as the **square** of the speed, v.

$R = 50$ when $v = 10$.

Find R when $v = 16$.

$R \propto v^2$ | $50 = k(10)^2$ | $R = \frac{1}{2}(16)^2$

$R = kv^2$ | $50 = 100k$ | $= \frac{1}{2} \times 256$

$k = \frac{1}{2}$ | $= 128$ [3]

Question 7

The wavelength, w, of a radio signal is inversely proportional to its frequency, f.
When $f = 200$, $w = 1500$.

(a) Find an equation connecting f and w.

$w \propto \frac{1}{f}$ | $1500 = \frac{k}{200}$ | $w = \frac{300{,}000}{f}$

$w = \frac{k}{f}$ | $k = 300{,}000$ [2]

(b) Find the value of f when $w = 600$.

$600 = \frac{300{,}000}{f}$

$f = 500$ [1]

Question 1

y is inversely proportional to $(x+1)^2$.
$y = 50$ when $x = 0.2$.

(a) Write y in terms of x. [2]

$y \propto \frac{1}{(x+1)^2}$

$y = \frac{k}{(x+1)^2}$

$50 = \frac{k}{(0.2+1)^2}$

$k = 72$

$y = \frac{72}{(x+1)^2}$

(b) Find the value of y when $x = 0.5$. [1]

$y = \frac{72}{(0.5+1)^2} = 32$

Question 2

h is directly proportional to the square root of p.
$h = 5.4$ when $p = 1.44$.

Find h when $p = 2.89$. [3]

$h \propto \sqrt{p}$

$h = k\sqrt{p}$

$5.4 = k\sqrt{1.44}$

$k = 4.5$

$h = 4.5\sqrt{2.89}$

$= 7.65$

Question 3

y is inversely proportional to $\sqrt{1+x}$.
When $x = 8$, $y = 2$.

Find y when $x = 15$. [3]

$y \propto \frac{1}{\sqrt{1+x}}$

$y = \frac{k}{\sqrt{1+x}}$

$2 = \frac{k}{\sqrt{1+8}}$

$k = 6$

$y = \frac{6}{\sqrt{1+15}}$

$= \frac{6}{4} = 1.5$

Question 4

y is inversely proportional to x^2.
When $x = 2$, $y = 8$.

Find y in terms of x. [2]

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2}$

$8 = \frac{k}{2^2}$

$k = 32$

$y = \frac{32}{x^2}$

Question 5

y is inversely proportional to x^2.
When $x = 5$, $y = 16$.

Find y when $x = 10$. [3]

$y \propto \frac{1}{x^2}$

$y = \frac{k}{x^2}$

$16 = \frac{k}{5^2}$

$k = 400$

$y = \frac{400}{10^2}$

$= \frac{400}{100}$

$= 4$

Question 6

y is directly proportional to the square root of $(x + 2)$.
When $x = 7$, $y = 2$.

Find y when $x = 98$. [3]

$y \propto \sqrt{x+2}$

$y = k\sqrt{x+2}$

$2 = k\sqrt{7+2}$

$k = \frac{2}{3}$

$y = \frac{2}{3}\sqrt{98+2}$

$= \frac{2}{3} \times 10$

$= \frac{20}{3}$

Question 7

d is inversely proportional to $(w + 1)^2$.
$d = 3.2$ when $w = 4$.

Find d when $w = 7$. [3]

$d \propto \frac{1}{(w+1)^2}$

$d = \frac{k}{(w+1)^2}$

$3.2 = \frac{k}{(4+1)^2}$

$k = 80$

$d = \frac{80}{(7+1)^2}$

$= \frac{80}{64}$

$= \frac{5}{4}$

Question 8

y is directly proportional to $(x + 2)^2$.
When $x = 8$, $y = 250$.

Find y when $x = 4$. [3]

$y \propto (x+2)^2$

$y = k(x+2)^2$

$250 = k(8+2)^2$

$k = \frac{250}{100} = 2.5$

$y = 2.5(4+2)^2$

$= 2.5 \times 36$

$= 90$

Question 9

t varies inversely as the square root of u.
$t = 3$ when $u = 4$.

Find t when $u = 49$. [3]

$t \propto \frac{1}{\sqrt{u}}$

$t = \frac{k}{\sqrt{u}}$

$3 = \frac{k}{\sqrt{4}}$

$k = 6$

$t = \frac{6}{\sqrt{49}}$

$= \frac{6}{7}$

Question 10

p varies directly as the square root of q.
$p = 8$ when $q = 25$.

Find p when $q = 100$. [3]

$p \propto \sqrt{q}$

$p = k\sqrt{q}$

$8 = k\sqrt{25}$

$k = \frac{8}{5}$

$p = \frac{8}{5}\sqrt{100}$

$= \frac{8}{5} \times 10$

$= 16$